The Golden Ratio

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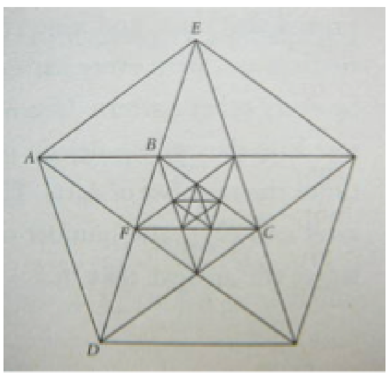
The intriguing Golden Ratio is a thread woven into two thousand years of mankind's history. The wonder of this number is not whether it can be found in the activities of man but that it underlies many of the works of nature to an astonishing degree. From the form of galaxies to the smallest sea creatures, from a falcon's flight pattern to horns, leaves and flowers, that unique number is all around us, buried in the building blocks of our world.

Given many names such as the Golden Number, the Golden Section, the Golden Mean, Divine Number and Divine Proportion or simply Phi (*fee*), it is said that it can be divined in botany, zoology, the human body, art, architecture, cosmology, metallic crystals, fractal systems, music, poetry, and even the stock market! Let's examine some of these instances and a little of its history.

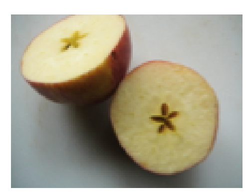
Almost certainly, a brotherhood led by **Pythagoras** (570–495BC), of right-triangle fame, discovered the Golden Ratio but the first person on record as actually calculating it is **Euclid** (ca. 300BC), the father of modern geometry. Its value is 1.618033988..., the dots meaning it is a never-ending, never-repeating decimal. In 1202, **Leonardo de Pisa** wrote a book in which he quotes a number of mathematical problems, one of which is especially significant in our story. Oddly, it concerns the rate at which rabbits reproduce: *A certain man put a pair of rabbits in a place surrounded on all sides by a wall. How many pairs of rabbits can be produced from that pair in a year if it is supposed that every month each pair begets a new pair which from the second month on becomes productive?* Month by month, the numbers lead to a series, as follows: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144.... **Johannes Kepler** (1571–1630), the astronomer who developed the first mathematical description of the solar system, noted that the ratios between consecutive terms in the series converge exactly to the Golden Ratio calculated by Euclid. Leonardo de Pisa's nickname was **Fibonacci** (son of Bonacci) and the series is now known as the *Fibonacci Series*. It seems inconceivable that a series derived from the procreation of rabbits could predict the ratio of male to female bees in a hive, yet it does just that, 1:1.618, the Golden Ratio, and it has yet more astonishing properties, as we shall see.



Mankind seems to be obsessed with five-pointed stars. They appear on over 60 national flags, and many logos such as Texaco's. We rate restaurants, hotels, services and a child's achievement with five-pointed stars. When asked to draw a star, most people draw a five-pointed one. The American military's insignia is a five-pointed star. If you join up the points of such a star you get a pentagon, a five-sided, geometric figure. The military's headquarters is The Pentagon (now there's a topic for the mystical conspiracy buffs).

Five is an important number for us; we have five fingers on each hand, five toes on each foot and five appendages to our torsos. It was equally important, if not more so, to the Pythagoreans back in 500BC; it was their number for love and marriage. They studied geometry including, among other things, the pentagram (five-pointed star), the pentagon (the five-sided shape), and dodecahedron (a solid with twelve pentagonal faces). They even used the pentagram as a badge and called it *Health*.

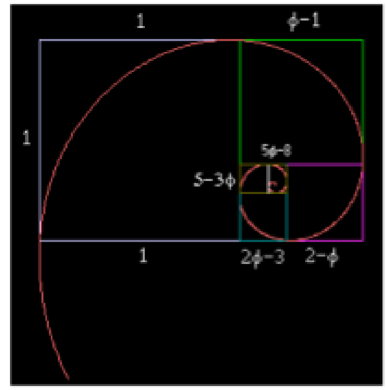
According to the writings of **Iamblichus** (245–325), it was a member of the sect, **Hippasus of Metapontum**, who discovered that the diagonals of the pentagon, which actually form the pentagram, intersect each other in precisely the Golden Ratio and that a side and a diagonal show the same ratio.

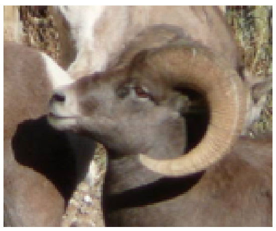


There are very many forms in nature that reflect the five-fold symmetry of the pentagon. In any guide to plants, five-petaled flowers predominate; join the points of the petals and you get a pentagon. There are starfish, sand dollars, okra and, of course, star fruit; even if you slice the humble apple across rather than vertically you will see a five-pointed star shape. All of these reflect the properties of the Golden Ratio.

The Nautilus is a South Pacific shellfish. As it grows, it constructs additional chambers in its shell of increasingly larger size into which it moves. Each increase in the length of the shell is accompanied by a proportionate increase in its radius so that the shape, though larger, remains unchanged and the growing Nautilus experiences an identical "home" throughout its lifetime. The series of chambers the Nautilus makes form a spiral, a *logarithmic (equiangular) spiral.* A property of this is that no matter where a radius is drawn from its center, it always meets the spiral at the same angle. The spiral's architecture has direct dimensional relationships with Phi.

Another distinguishing feature of logarithmic spirals is that the smallest part of the spiral, magnified and superimposed on the larger part, will fit it exactly. The mathematical name for this is *self-similarity* and it occurs in many natural features of all scales, like coastlines, mountain ranges, ferns and the veins of a leaf as well as in the pentagon. You've probably noticed that the drainage patterns in the mud at the side of a dirt road look just like river systems seen in photographs from space. Self-similarity is also a property of fractals to which Phi can be related.





The logarithmic spiral is unique in yet another way; it fits neatly inside a Golden Rectangle, a rectangle whose sides are in the proportion of 1:1.618.... This rectangle also possesses the property of self-similarity.

Nature is in love with logarithmic spirals. The spirals we see on the head of a sunflower, in tiny seashells, in hurricanes and even in the pattern formed when the water in your sink goes down the drain are all logarithmic spirals. So too, are the horns of rams, elephant tusks and some claws. Spiral galaxies are examples of logarithmic spirals on a cosmic scale.

Swiss mathematician, **Jakob Bernoulli** (1654–1705), became so obsessed with the properties and perfection of the logarithmic spiral that, in 1670, he wrote a book on the subject: *Spira Mirabilis*–the wonderful spiral. He even asked for one to be carved on his tombstone with the Latin text "*Eadem mutato resurgo*"– although changed, I rise again the same; a play on the properties of the spiral and on Christian resurrection beliefs. Jakob surely turned a spiral in his grave after his death because the mason responsible for his tomb erroneously carved on it the very different Archimedean spiral that can be seen at the botom of the memorial plaque.

In November 2000, **Vance Tucker**, a biologist at Duke University, published a paper on the attack pattern of birds of prey. He had observed that Peregrine Falcons do not dive directly toward their target, which would be the shortest distance, but spiral toward it. That seemed contrary to common sense as surely a straight line would be the fastest attack path.

Tucker theorized that it would be necessary for the bird to keep its "eye on the ball" as it attacked, but as the bird's eye is on the side of its head, it would need to cock its head about 40° to one side if it were to dive straight at the prey yet keep it in sight. His wind tunnel experiments proved that a substantial drag factor was generated once the head was turned at the necessary angle and that this would greatly reduce the bird's speed. Tucker's results show that Peregrines do indeed keep their head straight when diving and that they adopt a logarithmic spiral flight path to their prey. The equiangular properties of the spiral mean that they never have to take their eye off the target.

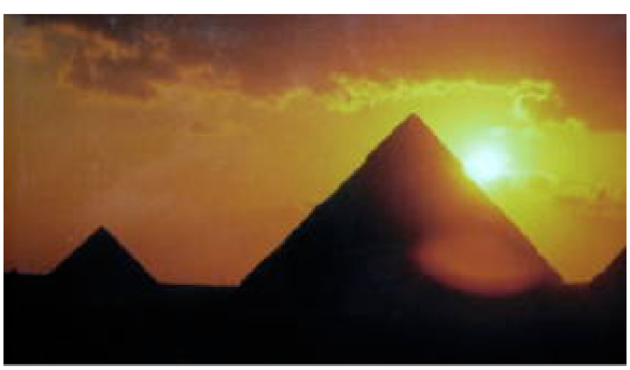
Theophrastus (372–287BC), **Pliny the Elder** (23–79), and **Leonardo da Vinci** (1452–1519) all noticed that leaves forming around a stem do so in a regular spiral pattern. Da Vinci even quantified it as 5 leaves every 2 turns. In botany, this is written as 2/5. Different trees have different ratios, for instance, Pussy Willow is 5/13, but they always turn out to be alternate terms in the Fibonacci series and are therefore closely related to Phi.

Dividing a circle, using reasoning similar to Euclid's, a Golden Angle can be calculated; its value is 137.5°. This is also significant in botany. In 1837, **Louis and Auguste Bravais** showed that when plants produce new leaves, they tend to advance around the stem of the plant in a tightly wound spiral pattern at an average angle of 137.5° to each other. At the time there was no explanation for this, but recent studies show that, for optimum exposure to sun and moisture, infinitely large leaf arrays indeed settle into this 137.5° pattern. Using the Golden Angle means that leaves never occur directly above each other on the stem. Leaf arrangements are therefore related to Phi in yet another way.

The flowers and seed heads of some plants, like the pinecone, coneflower or sunflower, can be considered very compressed stems that have large numbers of florets placed around them. If we look at a closed-up pinecone or the center of a sunflower, we see spiral patterns going both clockwise and counter-clockwise. You can trace the same thing on the surface of a pineapple and in certain cacti. The numbers of spirals exhibited in each direction are *always* consecutive terms of the Fibonacci series and therefore in the ratio of Phi. Studies show that this arrangement turns out to make the most efficient use of space on the seed head and its origin is judged to be biochemical.

Pick up any guide to flowers or the blossoms of trees and you will find that the majority of blooms have five petals, some three and some, like daisies or asters, have higher numbers. Few have petal numbers that are not in the Fibonacci series; for instance, lilies have 3, buttercups 5, marigolds 13 and asters 21. Most field daisies have 13, 21 or 34 petals. Even the beautiful arrangement of the petals of a rose is based on the Golden Ratio.

At this point in our story, it's right to sound a warning note. It's always possible in a sort of hindsight to superimpose a pattern that may not really be present, so it is important to consider what actual proof may also be available. In botany it is very clear, in other areas it is often not so. There are many websites that talk about the Golden Ratio and many, perhaps even most, are given to wild generalizations and unproven assertions. The mystical always holds more attraction than the scientific. Finding numerical relationships in the many dimensions of a building or painting, for instance, is not hard to do but this is quite different to proving that the designer consciously engineered them.



The pyramids of Egypt are a case in point. The most accurate dimensions available for the Great Pyramid of **Khufu** are Phi-related to an astonishing accuracy (less than 0.1% deviation). Unfortunately, this leads to some wild generalizations. There is a bald assertion on one website that *"Phi...has been used by mankind for centuries in architecture. Its use started as early as with the Egyptians in the design of the pyramids."* Let's examine this statement. The first recorded calculation of Phi was Euclid's in 300BC; Khufu's pyramid was built more than 2,000 years before that. In fact, of the many pyramids that have been discovered, this is the *only* one that exhibits Phi-related dimensions. Khufu's father, **Sneferu**, built the first pyramids and it took two unsuccessful attempts before he got one to stand up, so the form arose out of experimental error. Scholarly opinion notes that Egyptian mathematics had not developed to a standard that would permit them to calculate Phi; only one papyrus even refers to mathematics. However, there were several important subjects, like embalming, that were never written down. Could it have been a divine secret passed down verbally through the priesthood? It could – is it likely? Khufu's pyramid will remain a source of mystery that archeologists, scientists and mystics can argue over forever.

The burden of proof is never as crucial as in art. For instance, good circumstantial evidence indicates that Leonardo da Vinci knew about Phi since he illustrated friend **Luca Pacioli's** book, *Divina Proportione*, part of which eulogizes the Ratio, however, Da Vinci never refers to using it and no historical documents report that he did.

Salvador Dali did intentionally used the Golden Rectangle for his painting *"The Sacrament of the Last Supper"* and included part of a huge dodecahedron (the geometry of the pentagonal faces contains many instances of Phi). His thoughts may not have been wholly mystical, however. The painting references da Vinci's *"The Last Supper"* and the skeletal illustration of the dodecahedron that he drew for Pacioli's book. Da Vinci's painting is *not* in Golden Ratio format, being 1:1.93.

There is much talk of the Golden Rectangle being ***the*** aesthetically pleasing proportion in art and architecture. Recent experiments have, in fact, shown that there is no selective preference for the proportion of the Golden Rectangle over similar rectangles. It seems likely to me that the experienced artist's innate aesthetic intuition dictates proportion and composition in the paintings of the masters rather than any adherence to an arithmetical norm. Though it may approximate Phi, and that is remarkable in itself, it need not do so with any great accuracy to be pleasing.

The full story of the Golden Ratio is too diverse to cover in such a short article; I hope to post a series of articles that tell the full story later in the year. If you have an interest in knowledge for knowledge's sake, tracing the genealogy of Phi is a good way to satisfy it. As **Bertrand Russell** (1872–1970), a British philosopher, logician and essayist, once said: *"There is much pleasure to be gained from useless knowledge."*

If you have questions or comments about the Golden Ratio or this article please e-mail david@davidcuinart.com